

F-4/429/20

ROLL NO.

IV Semester Examination, May-June, 2020

M.Sc.
Mathematics
Paper IV

(Operation Research-II)

Time : 3 Hours

M.M. 80

Note: All Questions are Compulsory. Question Paper comprises of 3 sections. **Section A** is objective type/Multiple Choice questions with no internal Choice. **Sections B** is short answer type with internal Choice. **Section C** is long answer type with internal Choice.

SECTION 'A'

(Objective Type/Multiple Choice questions)

1x10=10

Choose the Correct Answer-

1. Bellman's Principle of optimality is used to solve
 - a) The transportation problem
 - b) The Dynamic programming problem
 - c) An Assignment problem
 - d) None of Above
2. The stages involved in solving an 'n' variable dynamic programming problem are –
 - a) n+1
 - b. n
 - c. n -1
 - d. None of the Above
3. A saddle point exists if
 - a) Maxmin value = Minimax value
 - b) Maximin value = Maximax value
 - c) Minimax value = Minimin value
 - d) None of the above

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4. Dominance principle in game theory
 - a) Reduces the size of the pay-off matrix
 - b) Reduces the game value
 - c) Both (a) and (b)
 - d) None of the Above
5. To solve an All integers programming problem, the method used is
 - a) Branch and bound method
 - b) Cutting plane method
 - c) Both (a) and (b)
 - d) None is above
6. In mixed integers programming problem decision variable are required to be
 - a) All integer
 - b) All fractional
 - c) Some integer, and some fractional
 - d) None of these
7. An Non linear programming problem in which the objective function is quadratic and constraints are linear is called-
 - a) A quadratic programming problem
 - b) A separable programming problem
 - c) Both (a) and (b)
 - d) None of these
8. Quadratic programming techniques provide solution to an NLPP-
 - a) Exact
 - b) Approximate
 - c) Some times exact and some times approximate
 - d) None of these
9. Input -output economic Analysis was developed by
 - a) Wassily Leontief
 - b) Beal's
 - c) King fisher
 - d) None of these

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10. Which of the following has maximum hydrogen/Carbon Ration (by weight)?

- a) Naphtha
- b) Gasoline
- c) Diesel
- d) Fuel Oil

SECTION 'B'

(Short Answer Type Questions)

4x5=20

Unit-I

1. Solve the following problem using Dynamic Programming.

$$\text{Min } Z = x_1^2 + x_2^2 + x_3^2$$

Subject to

$$x_1 + x_2 + x_3 \geq 15$$

$$\text{and } x_1 + x_2 + x_3 \geq 0$$

Or

Use the Principle of Optimality to find the maximum value of

$$b_1x_1 + b_2x_2 + b_3x_3 + \dots + b_nx_n$$

When

$$x_1 + x_2 + x_3 + \dots + x_n = c$$

$$x_i \geq 0, i = 1, 2, 3, \dots, n$$

Unit-II

2. Define the following

- (i) Saddle Point
- (ii) Solution of a game

Or

Let $\{V_{ij}\}$ be the pay-off matrix for a two person zero sum game.

If \underline{v} denotes the maxmin value and \bar{v} the minimax value of the game then prove. $\underline{v} \leq \bar{v}$

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Unit-III

3. Find the optimum all integers solution to the following IPP

$$\text{Max } z = x_1 + 2x_2$$

Subject to

$$2x_2 \leq 7$$

$$x_1 + x_2 \leq 7$$

$$2x_1 \leq 11,$$

$$x_1, x_2 \geq 0 \text{ and integers}$$

Or

Solve the following Mixed integers programming problem.

$$\text{Max } z = x_1 + 2x_2$$

Subject to

$$3x_1 + 2x_2 \leq 5$$

$$x_2 \leq 2$$

$$x_1, x_2 \geq 0 \text{ and } x_1 \text{ is an integers}$$

Unit-IV

4. Write short notes on blending problems.

Or

What is Input-Output Analysis?

Unit-V

5. Obtain the set of necessary Conditions for the Nonlinear programming problem.

$$\text{Min } Z_1 = 3x_1^2 + x_2^2 + 2x_1x_2 + 6x_1 + 2x_2$$

Subject to the Constraints.

$$2x_1 - x_2 = 4, \quad x_1, x_2 \geq 0$$

Or

Find the necessary conditions for maximum or minimum of the function.

$$Z_1 = f(X) = f(X_1, X_2, \dots, X_n)$$

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Subject to

$$g_1(x_1, x_2, \dots, x_n) = b_1$$

$$g_2(x_1, x_2, x_3, \dots, x_n) = b_2$$

$$x_1, x_2, \dots, x_n \geq 0, \quad b_1, b_2 \text{ are constants}$$

SECTION 'C'

(Long Answer Type Questions)

10x5=50

Unit-I

1. Use dynamic programming to solve

$$\text{Max } Z = 500x_1 + 800x_2$$

Subject to

$$5x_1 + 6x_2 \leq 60$$

$$x_1 + 2x_2 \leq 16$$

$$x_1 \leq 8$$

$$x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

Or

Solve by using Dynamic Programming

$$\text{Max } Z = x_1 + 9x_2$$

Subject to

$$2x_1 + x_2 \leq 25$$

$$x_2 \leq 11$$

$$x_1, x_2 \geq 0$$

Unit-II

2. Solve the game Graphically whose pay-off matrix is

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B

I II

| | | | |
|---|-----|----|---|
| A | I | 2 | 7 |
| | II | 3 | 5 |
| | III | 11 | 2 |

Or

For any zero sum two person game where the optimal strategies are not pure strategies and for which A pay off matrix is

B

| | | | |
|---|-----------|----------|-----------|
| | | $I(y_1)$ | $II(y_2)$ |
| A | $I(x_1)$ | a_{11} | a_{12} |
| | $II(x_2)$ | a_{21} | a_{22} |

Prove that the optimal strategies (x_1, x_2) and (y_1, y_2) given by

$$\frac{x_1}{x_2} = \frac{a_{22} - a_{21}}{a_{11} - a_{12}}, \quad \frac{y_1}{y_2} = \frac{a_{22} - a_{12}}{a_{11} - a_{21}}$$

and the value of the game to A is given by

$$v = \frac{a_{11}a_{22} - a_{12} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

Unit-III

3. Find the optimum integers solution to the following integers Programming problem.

$$\text{Max } Z = 2x_1 + 20x_2 - 10x_3$$

Subject to the Constraints

$$2x_1 + 20x_2 - 4x_3 \leq 15$$

$$6x_1 + 20x_2 + 4x_3 = 20$$

And $x_1, x_2, x_3 \geq 0$ and integer

Or

Solve the following MIPP.

$$\text{Max } Z = 4x_1 + 6x_2 + 2x_3$$

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Subject to

$$4x_1 - 4x_2 \leq 5$$

$$-x_1 + 6x_2 \leq 5$$

$$-x_1 + x_2 + x_3 \leq 5$$

$$x_1, x_2, x_3 \geq 0 \text{ and } x_2 \text{ is an integer.}$$

Unit-IV

4. Explain in brief Petroleum refinery operations.

Or

Explain how to Determine an optimal product mix?

Unit-V

5. Apply Wolfe's method to solve the following quadratic programming problem.

$$\text{Max } Z = 2x_1 + x_2 - x_1^2$$

Subject to

$$2x_1 + 3x_2 \leq 6$$

$$2x_1 + x_2 \leq 4,$$

$$\text{and } x_1, x_2 \geq 0$$

Or

Solve the following NLPP using the method of Lagrangian multipliers method.

$$\text{Min } Z = x_1^2 + x_2^2 + x_3^2$$

Subject to

$$4x_1 + x_2^2 + 2x_3 = 14,$$

$$x_1, x_2, x_3 \geq 0,$$

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